Minimum wetting rate for a decelerating liquid film

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Received April 1987 and accepted for publication February 1988

The paper presents an analytical study of a laminar decelerating liquid film falling along a vertical plate. Approximate solutions are obtained for the boundary layer within the film, film thickness, entrance length, and minimum wetting rate. It is shown that the analysis may be extended also to a film flowing over a horizontal cylinder. The theory is in reasonable agreement with the parametric trends observed in experiments on horizontal cylinders.

Keywords: film breakdown; minimum wetting rate; hydrodynamic entrance length

Introduction

The conditions under which a thin liquid film, driven along a solid surface by gravity or by shear stresses, breaks down into a series of rivulets, leaving dry patches on the solid surface, are of great importance in a number of technical applications. It is known that there is a minimum film thickness, and thus a minimum wetting rate, for a continuous film to be stable. The minimum film thickness depends on the shear stress, wall heat flux, contact angle, and physical properties. The problem of minimum film thickness has been studied by numerous authors. Generally, three different ways have been followed.

One approach to the formation of dry patches on the wall is via a stability method[1] which imposes a small disturbance on the laminar steady flow of a liquid film. The solution yields conditions for the growth of small disturbances leading in consequence to a breakdown of the film. However, the stability studies all conclude that a film becomes more unstable as the flow rate increases. On the other hand, experimental studies have shown that the film breaks down only when the film flow rate is below a certain value.

Another approach is using a rivulet model incorporating a minimum energy requirement.[2,3] A transition from film flow to rivulets occurs when the energy per unit width is the same for both the film and the rivulets. An additional condition is required for the system to be stable if the total energy takes a local minimum with respect to the rivulet spacing.

A third approach to the breakdown of thin films considers the stability of a dry patch and is based on the equilibrium of forces acting at the upstream stagnation point of the dry patch.[4,5] The force balance involves the inertial force due to the bringing of the upstream liquid to rest and the surface tension force due to a nonzero contact angle

$$\int_0^{\delta_f} \rho u^2 \, dy = \sigma(1-\cos\theta) \tag{1}$$

Solution of the equation depends on the velocity profile $u(y)$. Assuming a laminar and developed film flow caused by the gravity force only, the velocity profile is given by the well-known formula

$$u = \frac{\rho g}{2\mu}(2y\delta_f - y^2) \tag{2}$$

Thus the mass flow rate per unit width of the film, i.e., the wetting rate, is

$$\Gamma = \rho \int_0^{\delta_f} u \, dy = \frac{\rho^3 g \delta_f^3}{3\mu} \tag{3}$$

The minimum film thickness is evaluated by substitution of Equation 2 into Equation 1:

$$\delta_{fm} = \left(\frac{15}{2}\right)^{1/5} \left[\frac{\sigma(1-\cos\theta)}{\rho}\right]^{1/5} \left(\frac{\mu}{\rho g}\right)^{2/5} \tag{4}$$

Using Equation 3 the minimum wetting rate is obtained as

$$\Gamma_m = 1.116[\sigma(1-\cos\theta)]^{3/5}\left(\frac{\mu\rho}{g}\right)^{1/5} \tag{5}$$

It may be shown[6] that this very simple model gives the minimum film thickness only about 10% greater than the second one based on the principle of minimum energy. Therefore, the last model will be applied in the further analysis, as more convenient in use.

The film breakdown theories briefly discussed above concern the fully developed film flow. Such flow requires a hydrodynamic entrance section, i.e., a section which allows the flow to attain the above specified conditions. When one considers the flow over a surface with the entrance length small compared to the total surface length, then the entrance effect may be neglected. In other cases the effect should be taken into account. An example of the latter is the film flow over a bundle of horizontal tubes, which occurs in many engineering apparatus, like condensers, cascade or evaporative coolers.

The problem considered in the present paper concerns flow of a film falling over the leading edge of a vertical surface, when the entrance effect is significant. An analysis is carried out to obtain the entrance length, the thicknesses of the boundary layer within the film and of the film, and also the minimum wetting rate.

Since the film breakdown analysis in the entrance region will be carried out via the force balance model,[4,5] commonly used for the developed flow, then the results of it will suffer the same limitations as the original model does.

One of them is a difficulty in predicting the contact angle, since measurements of it under film breakdown conditions are not available. It is therefore generally estimated,[3,4] as the one required in the theory, in order to reproduce the experimental data.

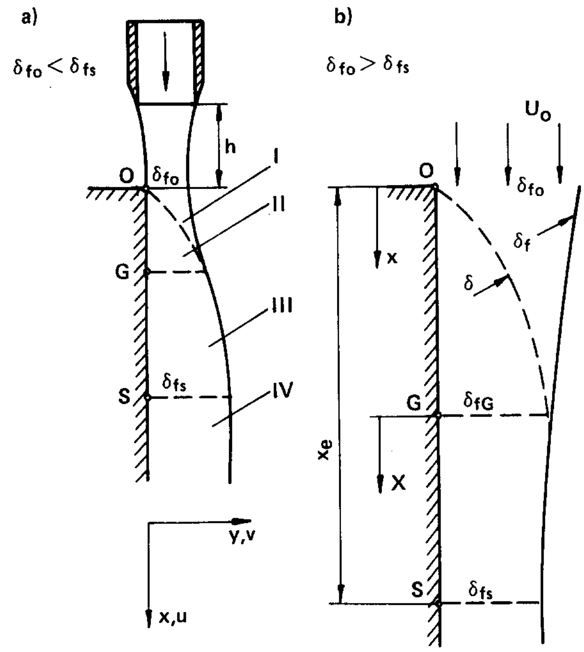

Figure 1 Flow situation in the entrance region

None of the existing film breakdown theories describes satisfactorily the effect of a wave motion in the film on its stability. On the other hand, experimental data[7] indicate that a very disturbed film breaks down more easily than the undisturbed one. The difference in the minimum wetting rates for both cases may even reach two orders of magnitude.

Analysis

A falling liquid film emerging from a slit or some other device and flowing over a vertical surface is considered here, as depicted in Figure 1. It is assumed that the flow is laminar, the free surface is waveless, and the shear stress at the gas-liquid interface is negligible. The film is considered to have initially the thickness δ_{f0} and a uniform velocity U_0.

For the above conditions one may distinguish two characteristic cases. Depending on the initial thickness, Figure 1, the film decelerates or accelerates to the limiting parabolic Nusselt velocity profile, as given by Equation 2. The former case is more important in a stability analysis since the thinner film is more vulnerable to breakdown. Generally, for both cases there are four regions of a flow domain: I—Inviscid region; II—boundary layer region; III—transition region; IV—fully developed flow region. The hydrodynamic entrance length comprises the second and third regions.

Inviscid region

In the first region the flow is inviscid and the velocity may be described by free-fall equations

$$U_0 = (U_{sl}^2 + 2gh)^{1/2} = (2gW)^{1/2} \quad (6)$$

at the leading edge and

$$U_\delta(x) = [2g(W+x)]^{1/2} \quad (6a)$$

at the edge of the boundary layer.

Fully developed flow region

For the fully developed flow region the velocity profile is described by Equation 2. Hence the pertinent parameters may be obtained as follows:

the mean film velocity:

$$\bar{u}_f = \frac{1}{\delta_{fs}} \int_0^{\delta_{fs}} u \, dy = \frac{\rho g}{3\mu} \delta_{fs}^2 \quad (7)$$

the volume flow rate per unit width:

$$q = \bar{u}_f \delta_{fs} = \frac{\rho g}{3\mu} \delta_{fs}^3 \quad (8)$$

Notation

d	Tube diameter
F_i	Inertial force
g	Gravitational acceleration
h	Distance from a slit
l	Length scale, Equation 22
t	Temperature
q	Volume flow rate per unit width
Re	Reynolds number, $\mathrm{Re} = 4\Gamma/\mu$
U_0	Film velocity at $x=0$
U_δ	Film velocity at $y=\delta$
U_f	Film velocity at $y=\delta_f$
$\bar{u}_f$	Mean film velocity
u, v	Velocities in x, y direction
x, y	Rectangular coordinates
x_e	Entrance length
X	Longitudinal coordinate in Figure 1
X^+	Dimensionless coordinate, Equation 40
W	Head of liquid above plate, $(h + U_{sl}^2/2g)^{1/2}$
α	Inclination angle of plate
δ	Boundary layer thickness
δ^+	Dimensionless boundary layer thickness, $\delta^+ = \delta/l$
δ_f	Film thickness
δ_f^+	Dimensionless film thickness, $\delta_f^+ = \delta_f/l$
Δ	Dimensionless film thickness, Equation 40
Γ	Mass flow rate per unit width, $\Gamma = \rho q$
μ	Dynamic viscosity
ρ	Density
σ	Surface tension
θ	Contact angle
τ	Shear stress
ξ	Dimensionless coordinate, $\xi = 1 + x/W$

Subscripts

ef	Effective
f	Film
G	Value at point G in Figure 1
h	Horizontal
m	Minimum
0	Initial
S	Value at point S in Figure 1
sl	Value at slit
x	x direction
w	Wall
δ	At boundary layer edge
∞	At infinity

the film thickness:

$$\delta_{fs}=\left(\frac{3\mu q}{\rho g}\right)^{1/3} \tag{9}$$

Boundary layer region

For standard boundary layer approximations for an incompressible fluid and neglecting pressure gradients, the governing equations of momentum and continuity take the form

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g-\frac{\mu}{\rho}\frac{\delta^2 u}{\delta y^2} \tag{10}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{11}$$

with boundary conditions

$$u=U_0=(2gW)^{1/2},\quad v=0,\qquad \text{at } x=0 \tag{12}$$

$$v=0,\quad \frac{\partial u}{\partial y}=\frac{\tau_w}{\mu}\qquad \text{at } y=0 \tag{13}$$

$$u=U_\delta(x),\quad v=v_\delta,\quad \frac{\partial u}{\partial y}=0\qquad \text{at } y=\delta \tag{14}$$

To solve the problem, Equation 10 is integrated with respect to y over the boundary layer thickness, and the velocity component $v(x, y)$ is eliminated from the resulting equation by means of the continuity equation. Since the flow outside the boundary layer is inviscid, the velocity $U_\delta(x)$ comes from the Euler equation

$$\frac{dU_\delta(x)}{dx}=\frac{g}{U_\delta(x)} \tag{15}$$

Therefore, the momentum integral equation is obtained as

$$\frac{d}{dx}\int_0^{\delta(x)} u(U_\delta-u)\,dy+\frac{dU_\delta}{dx}\int_0^{\delta(x)}(U_\delta-u)\,dy=\frac{\tau_w}{\rho} \tag{16}$$

To solve Equation 16 for the boundary layer thickness $\delta(x)$, one must assume the velocity profile within the boundary layer. For the case considered, the liquid film decelerates reaching the limiting parabolic velocity profile. It is therefore anticipated that a similar parabolic profile is arrived at very rapidly also within the boundary layer and remains "similar" throughout the flow field. This assumption is confirmed by the results of Stücheli and Ozisik.[8] With these considerations a velocity profile in the form

$$\frac{u}{U_\delta(x)}=2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2 \tag{17}$$

is chosen. Then the integrals and the wall shear stress term in Equation 16 become

$$\frac{d}{dx}\int_0^{\delta(x)} u(U_\delta-u)\,dy=\frac{2}{15}\frac{d}{dx}(\delta U_\delta^2) \tag{18}$$

$$\frac{dU_\delta}{dx}\int_0^{\delta(x)}(U_\delta-u)\,dy=\tfrac{1}{3}g\delta \tag{19}$$

$$\tau_w=\mu\left.\frac{\partial u}{\partial y}\right|_{y=0}=\frac{2\mu}{\delta}U_\delta \tag{20}$$

Substitution of Equations 18, 19, 20 into Equation 16 gives the differential equation for the boundary layer thickness:

$$\frac{d\delta}{dx}=\frac{1}{4g(W+x)}\left(\frac{30\mu[2g(W+x)]^{1/2}}{\rho\delta}-g_g\delta\right) \tag{21}$$

Introducing a characteristic length defined as

$$l=\left(\frac{U_0\mu}{\rho g}\right)^{1/2} \tag{22}$$

a dimensionless boundary layer thickness $\delta^+=\delta/l$, and a dimensionless coordinate $\xi=1+x/W$, we obtain the dimensionless differential equation

$$\frac{d\delta^+}{d\xi}+\frac{g\delta^+}{4\xi}=\frac{30}{4\xi^{1/2}\delta^+} \tag{23}$$

subject to the boundary condition $\delta^+=0$ at $\xi=1$. The solution of Equation 23 is the boundary layer thickness

$$\delta^+=\left[\frac{3(\xi^5-1)}{\xi^{9/2}}\right]^{1/2} \tag{24}$$

The film thickness $\delta_f(x)$ may be evaluated using the continuity equation in the form

$$\int_0^{\delta(x)} u\,dy+U_\delta(x)[\delta_f(x)-\delta(x)]=U_0\delta_{f0} \tag{25}$$

Using Equations 6 to 14, we write the equation for the liquid film thickness in the form

$$\delta_f^+=\frac{\delta_{f0}^+}{\xi^{1/2}}+\frac{\delta^+}{3} \tag{26}$$

The boundary layer region terminates at a point where the film thickness $\delta_f(x)$ and the boundary layer thickness $\delta(x)$ coincide (point G in Figure 1). Then according to Equation 26 the film thickness becomes

$$\delta_{fG}^+=\frac{3\delta_{f0}^+}{2\xi_G^{1/2}} \tag{27}$$

The distance required for the film to reach the point G may be obtained by comparing Equation 24 and Equation 27:

$$\xi_G=\left[\frac{4(\xi_G^5-1)}{3\delta_{f0}^{+2}}\right]^{2/7} \tag{28}$$

As can be seen, it is only the function of the initial film thickness δ_{f0}^+. The above equation was solved by the Newton method. The result is shown in Figure 2. By the method of least squares the distance ξ_G may be approximated also by the relation

$$\xi_G-1=0.21\delta_{f0}^{+2.25} \tag{29}$$

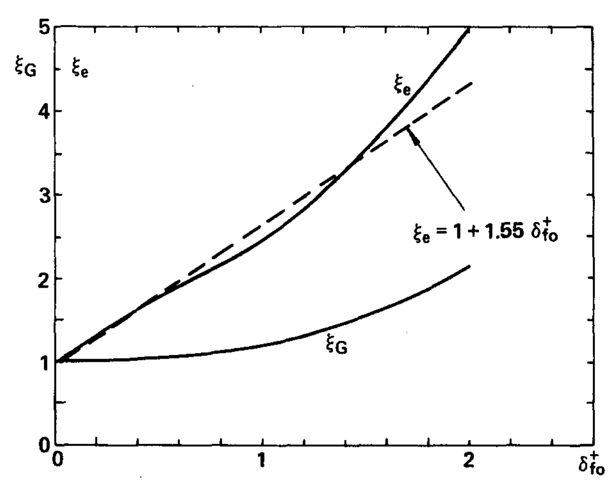

Figure 2 Dimensionless entrance lengths as functions of the initial film thickness

with the coefficient of determination $r^2=0.99$ in the range of δ_{f0}^+ from 0.1 to 2.

Transition region

The transition region is that region of the flow field in which the film thickness and the boundary layer thickness coincide, but the film thickness differs from its terminal value as given by Equation 2. The momentum integral equation, Equation 16, may be now rearranged to a more convenient form

$$\frac{d}{dx}\int_0^{\delta_f(x)} u^2\,dy - U_f\frac{d}{dx}\int_0^{\delta_f(x)} u\,dy = g\delta_f - \frac{\tau_w}{\rho} \tag{30}$$

In view of the continuity equation, Equation 25, the second term in Equation 30 vanishes. Thus the equation becomes

$$\frac{d}{dx}\int_0^{\delta_f(x)} u^2\,dy - g\delta_f = -\frac{\tau_w}{\rho} \tag{31}$$

The velocity profile is, as previously, assumed to be similar and parabolic, as given by Equation 17, with the exception that the surface velocity U_f is not given a priori, but has to be evaluated. The mean film velocity may be obtained with the aid of Equation 17 by exchanging U_f with U_δ:

$$\bar{u}_f = \frac{1}{\delta_f}\int_0^{\delta_f(x)} u\,dy = \tfrac{2}{3}U_f \tag{32}$$

Since the volume flow rate is given by

$$q = \bar{u}_f\delta_f = \tfrac{2}{3}U_f\delta_f \tag{33}$$

it follows that

$$U_f = \frac{3q}{2\delta_f} \tag{34}$$

$$\frac{dU_f}{dx} = -\frac{3q}{2\delta_f^2}\frac{d\delta_f}{dx} \tag{35}$$

and the respective terms of Equation 30 become

$$\frac{d}{dx}\int_0^{\delta_f(x)} u^2\,dy = \frac{4q}{5}\frac{dU_f}{dx} \tag{36}$$

$$\tau_w = \mu\left.\frac{\partial U}{\partial y}\right|_{y=0} = \frac{2\mu U_f}{\delta_f} \tag{37}$$

With the use of Equations 34 to 37 the differential equation

$$\frac{d\delta_f}{dx} = \frac{5}{6q^2}\left(\frac{3\mu q}{\rho} - g\delta_f^3\right) \tag{38}$$

is obtained describing the film thickness within the transition region. For the limiting case of $d\delta_f/dx \to 0$ the film thickness becomes

$$\delta_{f\infty} = \left(\frac{3\mu q}{\rho g}\right)^{1/3} \tag{39}$$

This is in agreement with the solution for the fully developed region, as given by Equation 9. Rewriting Equation 38 in terms of the dimensionless film thickness Δ and the dimensionless coordinate X^+ (Figure 1),

$$\Delta = \frac{\delta_f\rho q}{3\mu q}, \qquad X^+ = \frac{X\mu}{\delta_{fG}\rho q} \tag{40}$$

gives the dimensionless differential equation

$$\frac{d\Delta}{dX^+} = A\Delta^{2/3}(1-\Delta) \tag{41}$$

with boundary condition $\Delta=\Delta_0$ at $X^+=0$. Here

$$A = 7.5\Delta_0^{1/3} \qquad \text{and} \qquad \Delta_0 = \frac{\delta_{fG}\rho g}{3\mu q} \tag{42}$$

The solution of Equation 41 is

$$X^+ = \frac{1}{A}\left\{\frac{1}{2}\ln\left[\frac{1+\Delta^{1/3}+\Delta^{2/3}}{1+\Delta_0^{1/3}+\Delta_0^{2/3}}\left(\frac{1-\Delta_0^{1/3}}{1-\Delta^{1/3}}\right)^2\right] + \sqrt{3}\tan^{-1}\frac{2\Delta^{1/3}+1}{\sqrt{3}}\right\} - C \tag{43}$$

where

$$C = \frac{\sqrt{3}}{A}\tan^{-1}\frac{2\Delta_0^{1/3}+1}{\sqrt{3}}$$

It can be seen that the film reaches the limiting value $\Delta=1$ at $X^+\to\infty$. However, one can define the dimensionless length X_S^+ of the transient region, as to extend up to a location where Δ is within 1% of its limiting value. With this assumption the length X_S^+ is given by

$$X_S^+ = \frac{1}{A}\left\{\frac{1}{2}\ln\left[\frac{3}{1+\Delta_0^{1/3}+\Delta_0^{2/3}}\left(\frac{1-\Delta_0^{1/3}}{0.0034}\right)^2\right] + 1.812\right\} - C \tag{44}$$

and, according to Equation 40,

$$X_S = X_S^+\delta_{fG}\frac{\rho q}{\mu} \tag{45}$$

It is evident that the length of the transition region depends only on the initial film thickness δ_{f0}^+, since both Δ_0 and δ_{fG} depend on it alone.

Entrance length

The hydrodynamic entrance length is the sum of the lengths of the boundary layer region and the transition region

$$x_e = x_G + X_S \tag{46}$$

or, in dimensionless form,

$$\xi_e = 1 + \frac{x_e}{W} = \xi_G + \frac{X_S}{W} \tag{47}$$

The entrance length was calculated with the aid of Equations 28, 44 to 47 and is presented in Figure 2. It is convenient for the subsequent analysis to have an analytical form of Equation 47. Using the method of least squares leads to

$$\xi_e - 1 = 1.55\delta_{f0}^+ \tag{48}$$

or

$$x_e = 1.55W\delta_{f0}^+ \tag{49}$$

with the coefficient of determination $r^2=0.92$ in the range of δ_{f0}^+ from 0.1 to 2. The entrance length may also be expressed in terms of the Reynolds number, $\mathrm{Re}=4\Gamma/\mu$,

$$x_e = 1.55\frac{Wq}{lU_0} = 0.195l\mathrm{Re} \tag{50}$$

The laminar flow of the falling film is thought to exist over the Reynolds number range from 0 to 2000.[9]

It may be shown that result is related approximately to the range of the initial film thickness $\delta_{f0}^+<2$, if the head W is smaller than 5×10^{-2} m.

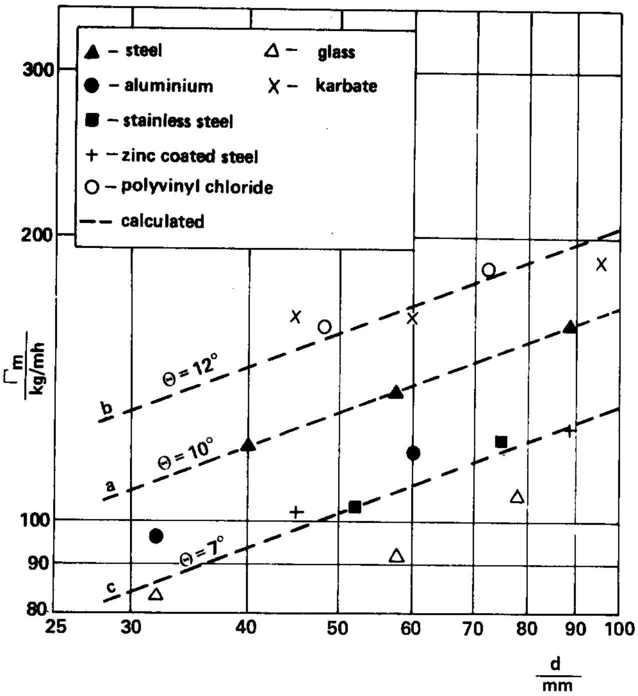

Figure 3 Comparison of experimental and theoretical minimum wetting rates

Minimum wetting rate

In the case of a decelerating liquid film falling over a vertical surface, the kinetic energy diminishes reaching its minimum value at the end of the entrance region, point S in Figure 1. The inertial force brought about by the loss in the downward momentum flux at the apex of the dry patch is equal to

$$F_i=\rho\int_0^{\delta_f} u^2\,dy=\tfrac{8}{15}\rho U_f^2\delta_f \tag{51}$$

if the velocity profile has the form of Equation 17. Making use of Equation 34, the inertial force at the point S may be written as

$$F_i=\frac{6\rho q^2}{5\delta_{fS}} \tag{52}$$

According to the "force model" of the film breakdown theories, this force has to be balanced by the surface tension

$$\frac{6\rho q_m^2}{5\delta_{fS}}=\sigma(1-\cos\theta) \tag{53}$$

Hence

$$q_m^2=\frac{5\sigma(1-\cos\theta)\delta_{fS}}{6\rho} \tag{54}$$

The film thickness should be associated with the flow parameters in the entrance region. Making use of Equation 50 leads to

$$q_m=\frac{2\mu x_e}{1.55 l\rho} \tag{55}$$

and according to Equation 7 and Equation 8, the mean film velocity may be expressed in terms of the volume flow rate:

$$\bar{u}_f=q^{2/3}\left(\frac{\rho g}{3\mu}\right)^{1/3} \tag{56}$$

Then

$$\delta_{fS}=\frac{q_m}{\bar{u}_f}=\frac{2\mu x_e}{1.55\rho l q_m^{2/3}(\rho g/3\mu)^{1/3}} \tag{57}$$

Substitution of Equation 57 into Equation 54 gives

$$q_m^{8/3}=1.075\frac{\sigma u}{\rho^2 l}\left(\frac{3\mu}{\rho g}\right)^{1/3}(1-\cos\theta)x_e \tag{58}$$

After some manipulations the minimum wetting rate may be written as

$$\Gamma_m=\rho q_m=1.03\left[\frac{10.125\rho^{10}\mu^{10}}{W^3 g}[\sigma(1-\cos\theta)x_e]^{12}\right]^{1/32} \tag{59}$$

A scrutiny of this result reveals that for a given liquid the minimum wetting rate is a strong function of the contact angle and the entrance length. Surprisingly, the influence of two other parameters, W and g, is much less pronounced, particularly the influence of the acceleration of gravity. The last conclusion is very important, since it allows the extension of the analysis to other configurations of laminar film flow, where slope is varying in the uniform gravity field. A film flow over a horizontal cylinder is an example.

The minimum wetting rate for the inclined plate is easily obtained from Equation 59 by replacing the gravity g by its component $g_x=g\sin\alpha$ acting along the plate. The slope of a horizontal cylindrical surface changes in the range $0\leqslant\alpha\leqslant\pi/2$. Therefore the minimum wetting rate for this surface may, in general, be written as

$$\Gamma_{mh}=1.03\left[\frac{10.125\rho^{10}\mu^{10}}{W^3 g\sin\alpha_{ef}}[\sigma(1-\cos\theta)x_e]^{12}\right]^{1/32} \tag{60}$$

where α_{ef} is the effective angle of inclination. It is calculated by making use of the mean value theorem. Thus

$$\left(\frac{1}{\sin\alpha_{ef}}\right)^{1/32}=\frac{2}{\pi}\int_0^{\pi/2}\frac{d\alpha}{(\sin\alpha)^{1/32}} \tag{61}$$

and by numerical integration one finds $\alpha_{ef}=39.5°$. Hence the ratio of the minimum wetting rate for the horizontal cylinder and the vertical plate is

$$\frac{\Gamma_{mh}}{\Gamma_m}=\frac{1}{(\sin\alpha_{ef})^{1/32}}=1.0142 \tag{62}$$

It can be seen that the difference in Γ_m for both configurations is small and may be neglected in a further analysis.

Comparison with experiments

There is no record in the open literature of any attempt to find analytically the minimum wetting rate in the entrance region. On the other hand, experimental investigations of film breakdown on horizontal tubes have been conducted and the results reported. Synowiec[10] has carried out the measurements of the minimum wetting rate on a tube bundle. Five horizontal tubes, imitating a cascade cooler, were arranged in the vertical column. Water as the test fluid was fed to a spray tube at the top of the bundle. Water from inside the spray tube flowed out through a series of holes and fell down onto the first tube, flowing around its circumference dropping onto the tube below.

The minimum wetting rate was examined in a range of the outside tube diameter from 32 to 95 mm, water temperatures from 10 to 15°C, and tube spacing from 0.3 to 60 mm. The experimental results of the minimum wetting rate for six different tube materials are presented in Figure 3. Synowiec[10] drew the following conclusions from his results:

1. The tube spacing has a negligible effect on the minimum wetting rate.
2. Wettability of the surface and the tube diameter play the dominant role. The influence of the first parameter, commonly expressed through the value of the contact angle, was shown by grouping the materials used into groups a, b, and c according to their wettability. For steel, representing the material of medium wettability, Synowiec has proposed a correlation

$$\frac{\Gamma_m}{\text{kg/mh}} = 30\left(\frac{d}{\text{mm}}\right)^{0.3734} \tag{63}$$

which is in agreement with experimental data to within $\pm 25\%$.
3. For the materials of groups b and c, the value of Γ_m should be increased or decreased by 25%, respectively, as compared with Equation 63.

A comparison between the Synowiec[10] results and those of the present analysis may be carried out with the aid of Equation 59. Evaluating the physical properties at a water temperature of $t = 15°C$, assuming an average water head $W = 25.4$ mm (1 inch) and tube diameter d equal to the entrance length x_e, one gets from Equation 59

$$\frac{\Gamma_m}{\text{kg/mh}} = 150\left[(1-\cos\theta)\left(\frac{d}{\text{mm}}\right)\right]^{0.375} \tag{64}$$

As is seen, a good qualitative agreement with respect to the exponents and the parameters involved in both formulas has been obtained. However, a quantitative comparison is not possible. The main reason for it is the lack of information about the contact angle, which was not measured in the experiments discussed. This is also a serious problem for all existing film breakdown theories concerning developed flow. There are a few data available on the contact angle under flow conditions, but only for the rivulets.[11,12] The contact angle is a complex and unknown function of surface and fluid properties and also of flow conditions. It is therefore treated generally as an adjustable constant in the models, in order to fit experimental data.[3,4] The same procedure is adopted in this paper. Assuming for the materals of the groups b, a, c in Figure 3 the values of the contact angle equal to 12°, 10°, and 7°, respectively, one gets from Equation 64

$$\frac{\Gamma_m}{\text{kg/mh}} = 35.9\left(\frac{d}{\text{mm}}\right)^{0.375} \quad \text{for b}$$
$$\frac{\Gamma_m}{\text{kg/mh}} = 30\left(\frac{d}{\text{mm}}\right)^{0.375} \quad \text{for a} \tag{65}$$
$$\frac{\Gamma_m}{\text{kg/mh}} = 23.5\left(\frac{d}{\text{mm}}\right)^{0.375} \quad \text{for c}$$

For a contact angle of 10° there is excellent agreement between Equation 65 and the Synowiec correlation, Equation 63. The data of Towell and Rathfeld[11] indicate the value of the contact angle from 4.5° to 12° for water flowing over a glass plate.

The experimental data of Ganic and Roppo[13] concern the film breakdown on a heated horizontal tube, caused by the thermal effect, i.e., the applied heat flux. It is therefore quite a different case than that considered here. However, it is interesting to mention that also in that case the tube spacing plays a small role.

Conclusions

The theoretical analysis presented, demonstrates that the minimum wetting rate for a decelerating liquid film falling over a surface is primarily a function of the surface size and the contact angle. The theoretical results are in quite reasonable agreement with data obtained for a bundle of horizontal tubes.

Acknowledgment

The author wishes to thank Professor J. Mikielewicz for useful discussions.

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